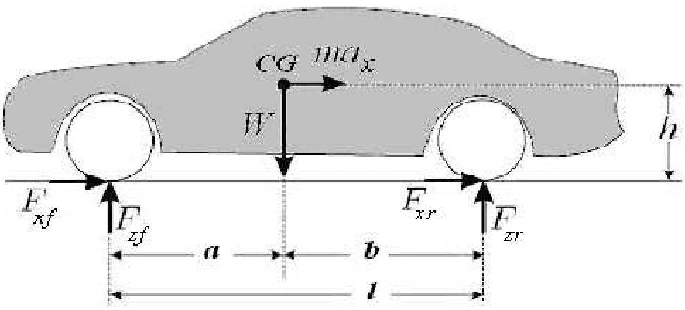
Vehicle Ride and Handling

Maplesoft, February, 2013



Consider a car with the following specifications:
Vehicle curb weight (kg):
$m_{curb} := 1000$:

Total vehicle sprung mass weight (kg): $m_{sprung} := 900$:

Unsprung weight per wheel, front and rear (kg): $m_{unsprungF} := 2 \cdot 25$: $m_{unsprungR} := 2 \cdot 25$:

Vehicle inertia about z axis ($kg \cdot m^2$): $I_z := 2000$:

Sprung mass inertial about x axis ($kg \cdot m^2$): $I_x := 750$:

Wheel base (m):
$L_{wb} := 2.5$:

Track (front and rear) (Ts m): $L_{track} := 1.4$:

Distance of CG from front axle (a m): $a_{CG} := 1.2$:

Distance of CG from rear axle (b m): $b_{CG} := 1.3$:

Vehicle CG height =0.6 m: $h_{CG} := 0.6$:

Sprung mass CG height = 0.7m: $h_{sprung} := 0.7$:

Roll center height (front/rear) = 0.2m: $h_{RC} := 0.2$:

Tire cornering stiffness (front and rear)= 25000 N/rad:
$C_\alpha := 25000$:

Tire vertical stiffness = 150,000 N/m: $C_z := 150000$:

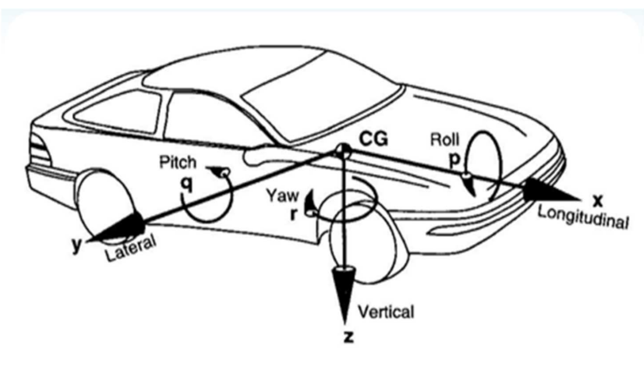
Tire camber stiffness (front and rear)=5000 N/rad: $C_\gamma := 5000$:

Front shock absorber rate (per wheel) = 800 N/m/s: $C_{df} := 800$:

Rear shock absorber rate (per wheel) = 1000 N/m/s: $C_{dr} := 1000$:

Distance between installation point of Left and right springs (front and rear) = 1.3 m: $T_s := 1.3$:

Distance between installation point of Left and right shock absorber (front and rear)= 1.3 m: $T_d := 1.3$:
Installation factor for springs and shock absorbers for front and rear wheels = 1:



Requirements

o The first natural frequency of front suspension (Hz)

$F_{nFront} := 1.0 :$

$\omega_{nFront} := F_{nFront} \cdot 2\,\pi = 6.283185308$

o The first natural frequency of rear suspension (Hz)

$F_{nRear} := 1.2 :$

$\omega_{nRear} := F_{nRear} \cdot 2\,\pi = 7.539822370$

Anti-roll-bar on front axle, equivalent torsional stiffness of the anti-roll-bar for:

Roll Gain in deg/g: $R_{gain} := 4 :$

Consider quarter car model, natural frequency of sprung mass is given by:

$$OmegaN := \omega_n = \sqrt{\frac{k_s}{m_s}} :$$

where m_s is the sprung mass.

For the front wheels:

$$m_{sF} := \frac{a_{CG}}{L_{wb}} \cdot m_{curb} - m_{unsprungF} = 430.0000000$$

Hence, the spring stiffness per wheel at the front to give the required natural frequency:

$$k_{sF} := solve\left(eval\left(OmegaN, \left[\omega_n = \omega_{nFront}, m_s = \frac{m_{sF}}{2} \right] \right), k_s \right) = 8487.859786$$

For the rear wheels:

$$m_{sR} := \frac{b_{CG}}{L_{wb}} \cdot m_{curb} - m_{unsprungR} = 470.0000000$$

Hence, the spring stiffness per wheel at the rear to give the required natural frequency:

$$k_{sR} := solve\left(eval\left(OmegaN, \left[\omega_n = \omega_{nRear}, m_s = \frac{m_{sR}}{2} \right] \right), k_s \right) = 13359.49652$$

General expression for the Roll Gain is:

$$RollGain := \frac{|\Phi_{ss}|}{a_y} = \frac{m_s \cdot h}{k_t - m_s \cdot g \cdot h} :$$

Roll gain in rad/m/s^2: $R_{gain} := \dfrac{R_{gain} \cdot \pi}{180 \cdot g} = 0.007116531100$

$$K_t := solve\left(eval\left(rhs(RollGain), \left[h = \left(h_{CG} - h_{RC} \right), m_s = m_{sprung} \right] \right) = R_{gain}, k_t \right) = 54118.04372$$

Therefore the equivalent torsional stiffness of the anti-roll bar is given by:

$$solve\left(K_t = \frac{1}{2} \cdot T_s^2 \cdot k_{sF} + \frac{1}{2} \cdot T_s^2 \cdot k_{sR} + K_{tARBf}, K_{tARBf} \right) = 35657.02764$$

Equivalently, the torsional damping can be found from..

$$TorsionalDamping := c_t = \frac{1}{2} T_{csF}^{\ 2} \cdot c_{sF} + \frac{1}{2} T_{csR}^{\ 2} \cdot c_{sR} :$$

$$C_t := eval\left(rhs(TorsionalDamping), \left[T_{csF} = T_d, T_{csR} = T_d, c_{sF} = C_{df}, c_{sR} = C_{dr} \right] \right) = 1521.000000$$

$$K_{us3DOF} := \frac{\left(m_s \cdot h_{roll} \cdot \left(K_{SBRF} - K_{SBRR} \right) \right)}{K_t - m_s \cdot g \cdot h_{roll}} + \left(\frac{m_s \cdot h_{roll} \cdot \left(K_{CBRF} \cdot \frac{C_{\gamma F}}{C_{\alpha F}} - K_{CBRR} \cdot \frac{C_{\gamma R}}{C_{\alpha R}} \right)}{K_t - m_s g \cdot h_{roll}} \right) + \left(\frac{-m_c \left(a \cdot C_{\alpha F} - b \cdot C_{\alpha R} \right)}{2 \cdot L_{wb} \cdot C_{\alpha F} \cdot C_{\alpha R}} \right) :$$

$$\begin{aligned} YawGain := &-\left(2.\, u(t)\, C_{\alpha F}\, C_{\alpha R} \left(981.\, a\, h_{roll}\, m_s + 981.\, b\, h_{roll}\, m_s - 100.\, a\, k_t - 100.\, b\, k_t\right)\right) \Big/ \left(200.\, K_{SBRF}\, u(t)^2\, h_{roll}\, m_s\, a\, C_{\alpha F}\, C_{\alpha R} - 200.\, K_{CBRR}\, u(t)^2\, h_{roll}\, m_s\, b\, C_{\gamma R}\, C_{\alpha F} \right. \\ &- 200.\, K_{SBRR}\, u(t)^2\, h_{roll}\, m_s\, b\, C_{\alpha R}\, C_{\alpha F} - 200.\, C_{\alpha F}\, a\, C_{\gamma R}\, K_{CBRR}\, u(t)^2\, h_{roll}\, m_s - 200.\, C_{\alpha F}\, a\, K_{SBRR}\, u(t)^2\, h_{roll}\, m_s\, C_{\alpha R} + 200.\, C_{\alpha R}\, b\, C_{\gamma F}\, K_{CBRF}\, u(t)^2\, h_{roll}\, m_s \\ &+ 200.\, C_{\alpha R}\, b\, K_{SBRF}\, u(t)^2\, h_{roll}\, m_s\, C_{\alpha F} + 200.\, a^2\, k_t\, C_{\alpha F}\, C_{\alpha R} + 200.\, b^2\, k_t\, C_{\alpha F}\, C_{\alpha R} - 100.\, C_{\alpha F}\, a\, u(t)^2\, k_t\, m_c + 400.\, C_{\alpha F}\, a\, b\, k_t\, C_{\alpha R} + 100.\, C_{\alpha R}\, b\, u(t)^2\, k_t\, m_c \\ &- 1962.\, a^2\, h_{roll}\, m_s\, C_{\alpha F}\, C_{\alpha R} - 1962.\, b^2\, h_{roll}\, m_s\, C_{\alpha F}\, C_{\alpha R} + 200.\, K_{CBRF}\, u(t)^2\, h_{roll}\, m_s\, a\, C_{\gamma F}\, C_{\alpha R} + 981.\, C_{\alpha F}\, a\, u(t)^2\, h_{roll}\, m_c\, m_s - 3924.\, C_{\alpha F}\, a\, b\, h_{roll}\, m_s\, C_{\alpha R} \\ &\left. - 981.\, C_{\alpha R}\, b\, u(t)^2\, h_{roll}\, m_c\, m_s \right) : \end{aligned}$$

Vehicle Parameters

$$CarParams := \left[a = a_{CG}, b = b_{CG}, l_{wb} = L_{wb}, m_c = m_{curb}, m_s = m_{sprung}, h_{roll} = h_{CG} - h_{RC}, Izz = I_z, Ixx = I_x, c_t = C_t, k_t = K_t \right]$$

$$\left[a = 1.2, b = 1.3, l_{wb} = 2.5, m_c = 1000, m_s = 900, h_{roll} = 0.4, Izz = 2000, Ixx = 750, c_t = 1521.000000, k_t = 54118.04372 \right] \quad (1.1$$

$$TireParams := \left[C_{\alpha F} = 2\, C_\alpha, C_{\alpha R} = 2\, C_\alpha, C_{\gamma F} = 2\, C_\gamma, C_{\gamma R} = 2\, C_\gamma \right]$$

$$\left[C_{\alpha F} = 50000, C_{\alpha R} = 50000, C_{\gamma F} = 10000, C_{\gamma R} = 10000 \right] \quad (1.2$$

Neutral-Steer Response

The vehicle's steering response is determined to be "neutral" when $K_{us3DOF} = 0$, so keeping all but one of the roll parameters constant, it is possible to determine the value of a roll parameter (in this case, K_{SBRF}) to provide a neutral response. This can then be used to plot with other steering responses to provide a guideline as to how far from neutral the response is.

Value of K_{SBRF} at neutral steer...

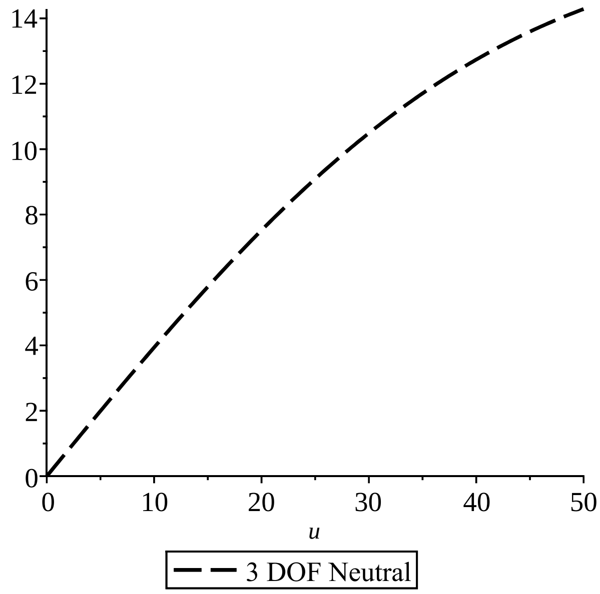
$KNeutral_{SBRF} := solve\left(eval\left(K_{us3DOF} = 0, \left[op(CarParams), op(TireParams), K_{SBRR} = 0.1, K_{CBRF} = 1, K_{CBRR} = 0.1\right]\right), K_{SBRF}\right)$

$$-0.07998875854 \qquad (1$$

$NeutralParams := \left[K_{SCBRF} = 0, K_{SCBRR} = 0, K_{SBRR} = 0.1, K_{CBRF} = 1, K_{CBRR} = 0.1, K_{SBRF} = KNeutral_{SBRF}\right]$

$$\left[K_{SCBRF} = 0, K_{SCBRR} = 0, K_{SBRR} = 0.1, K_{CBRF} = 1, K_{CBRR} = 0.1, K_{SBRF} = -0.07998875854\right] \qquad (2$$

$YawGain3DOFNeutral := plot(eval(YawGain, [op(CarParams), op(TireParams), op(NeutralParams)]), u = 0..50, colour = black, thickness = 2, linestyle = dash, legend = \text{"3 DOF Neutral"}) :$



This interactive tool allows the user to try various combinations of steer- and camber-by-roll coefficients and observe the effect on the yaw gain curve and the value of th understeer coefficient, K_{us}.