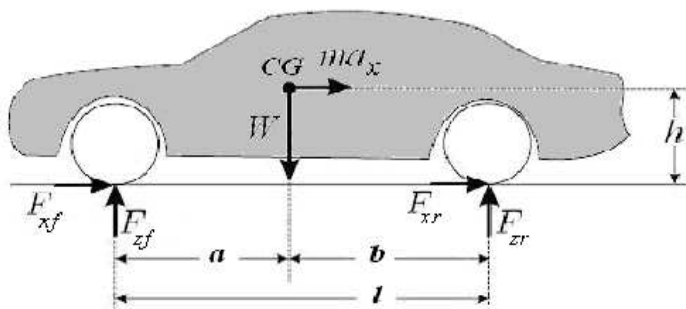


Vehicle Ride and Handling

Maplesoft, February, 2013



Consider a car with the following specifications:

Vehicle curb weight (kg):

$$m_{curb} := 1000 :$$

Total vehicle sprung mass weight (kg): $m_{sprung} := 900 :$

Unsprung weight per wheel, front and rear (kg): $m_{unsprungF} := 2.25 : m_{unsprungR} := 2.25 :$

Vehicle inertia about z axis ($kg \cdot m^2$): $I_z := 2000 :$

Sprung mass inertial about x axis ($kg \cdot m^2$): $I_x := 750 :$

Wheel base (m):

$$L_{wb} := 2.5 :$$

Track (front and rear) (Ts m): $L_{track} := 1.4 :$

Distance of CG from front axle (a m): $a_{CG} := 1.2 :$

Distance of CG from rear axle (b m): $b_{CG} := 1.3 :$

Vehicle CG height =0.6 m: $h_{CG} := 0.6 :$

Sprung mass CG height = 0.7m: $h_{sprung} := 0.7 :$

Roll center height (front/rear) = 0.2m: $h_{RC} := 0.2 :$

Tire cornering stiffness (front and rear)= 25000 N/rad:

$$C_{\alpha} := 25000 :$$

Tire vertical stiffness = 150,000 N/m: $C_z := 150000 :$

Tire camber stiffness (front and rear)=5000 N/rad: $C_{\gamma} := 5000 :$

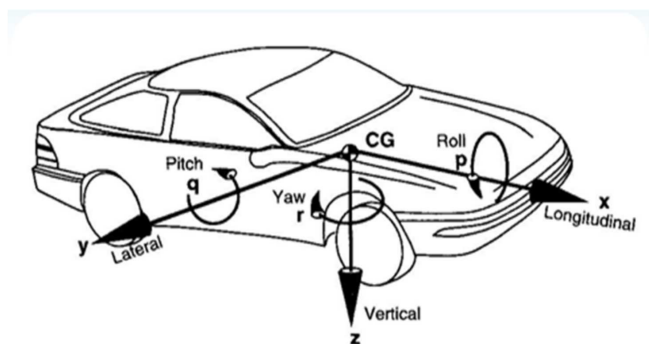
Front shock absorber rate (per wheel) = 800 N/m/s: $C_{df} := 800 :$

Rear shock absorber rate (per wheel) = 1000 N/m/s: $C_{dr} := 1000 :$

Distance between installation point of Left and right springs (front and rear) = 1.3 m: $T_s := 1.3 :$

Distance between installation point of Left and right shock absorber (front and rear)= 1.3 m: $T_d := 1.3 :$

Installation factor for springs and shock absorbers for front and rear wheels = 1:



Requirements

o The first natural frequency of front suspension (Hz)

$$F_{nFront} := 1.0 :$$

$$\omega_{nFront} := F_{nFront} \cdot 2 \pi = 6.283185308$$

o The first natural frequency of rear suspension (Hz)

$$F_{nRear} := 1.2 :$$

$$\omega_{nRear} := F_{nRear} \cdot 2 \pi = 7.539822370$$

Anti-roll-bar on front axle, equivalent torsional stiffness of the anti-roll-bar for:

$$\text{Roll Gain in deg/g: } R_{gain} := 4 :$$

Consider quarter car model, natural frequency of sprung mass is given by:

$$\text{OmegaN} := \omega_n = \sqrt{\frac{k_s}{m_s}} :$$

where m_s is the sprung mass.

For the front wheels:

$$m_{sF} := \frac{a_{CG}}{L_{wb}} \cdot m_{curb} - m_{unsprungF} = 430.0000000$$

Hence, the spring stiffness per wheel at the front to give the required natural frequency:

$$k_{sF} := \text{solve}\left(\text{eval}\left(\text{OmegaN}, \left[\omega_n = \omega_{nFront}, m_s = \frac{m_{sF}}{2}\right]\right), k_s\right) = 8487.859786$$

For the rear wheels:

$$m_{sR} := \frac{b_{CG}}{L_{wb}} \cdot m_{curb} - m_{unsprungR} = 470.0000000$$

Hence, the spring stiffness per wheel at the rear to give the required natural frequency:

$$k_{sR} := \text{solve}\left(\text{eval}\left(\text{OmegaN}, \left[\omega_n = \omega_{nRear}, m_s = \frac{m_{sR}}{2}\right]\right), k_s\right) = 13359.49652$$

General expression for the Roll Gain is:

$$\text{RollGain} := \frac{|\Phi_{ss}|}{a_y} = \frac{m_s \cdot h}{k_t - m_s \cdot g \cdot h} :$$

$$\text{Roll gain in rad/m/s}^2: R_{gain} := \frac{R_{gain} \cdot \pi}{180 \cdot g} = 0.007116531100$$

$$K_t := \text{solve}\left(\text{eval}(\text{rhs}(\text{RollGain}), [h = (h_{CG} - h_{RC}), m_s = m_{sprung}])\right) = R_{gain}, k_t) = 54118.04372$$

Therefore the equivalent torsional stiffness of the anti-roll bar is given by:

$$\text{solve}\left(K_t = \frac{1}{2} \cdot T_s^2 \cdot k_{sF} + \frac{1}{2} \cdot T_s^2 \cdot k_{sR} + K_{IARBf} \cdot K_{IARBf}\right) = 35657.02764$$

Equivalently, the torsional damping can be found from..

$$\text{TorsionalDamping} := c_t = \frac{1}{2} T_{csF}^2 \cdot c_{sF} + \frac{1}{2} T_{csR}^2 \cdot c_{sR} :$$

$$C_t := \text{eval}(\text{rhs}(\text{TorsionalDamping}), [T_{csF} = T_d, T_{csR} = T_d, c_{sF} = C_{dF}, c_{sR} = C_{dR}]) = 1521.000000$$

$$K_{us3DOF} := \frac{(m_s \cdot h_{roll} \cdot (K_{SBRF} - K_{SBR}))}{K_t - m_s \cdot g \cdot h_{roll}} + \left(\frac{m_s \cdot h_{roll} \cdot \left(K_{CBRF} \cdot \frac{C_{\gamma F}}{C_{\alpha F}} - K_{CBRR} \cdot \frac{C_{\gamma R}}{C_{\alpha R}} \right)}{K_t - m_s \cdot g \cdot h_{roll}} \right) + \left(\frac{-m_c \cdot (a \cdot C_{\alpha F} - b \cdot C_{\alpha R})}{2 \cdot L_{wb} \cdot C_{\alpha F} \cdot C_{\alpha R}} \right) :$$

$$\text{YawGain} := - \left(2 \cdot u(t) \cdot C_{\alpha F} \cdot C_{\alpha R} \left(981 \cdot a \cdot h_{roll} \cdot m_s + 981 \cdot b \cdot h_{roll} \cdot m_s - 100 \cdot a \cdot k_t - 100 \cdot b \cdot k_t \right) \right) / \left(200 \cdot K_{SBRF} \cdot u(t)^2 \cdot h_{roll} \cdot m_s \cdot a \cdot C_{\alpha F} \cdot C_{\alpha R} - 200 \cdot K_{CBRR} \cdot u(t)^2 \cdot h_{roll} \cdot m_s \cdot b \cdot C_{\gamma R} \cdot C_{\alpha F} \right. \\ \left. - 200 \cdot K_{SBRR} \cdot u(t)^2 \cdot h_{roll} \cdot m_s \cdot b \cdot C_{\alpha R} \cdot C_{\alpha F} - 200 \cdot C_{\alpha F} \cdot a \cdot C_{\gamma R} \cdot K_{CBRR} \cdot u(t)^2 \cdot h_{roll} \cdot m_s - 200 \cdot C_{\alpha F} \cdot a \cdot K_{SBRR} \cdot u(t)^2 \cdot h_{roll} \cdot m_s \cdot C_{\alpha R} + 200 \cdot C_{\alpha R} \cdot b \cdot C_{\gamma F} \cdot K_{CBRF} \cdot u(t)^2 \cdot h_{roll} \cdot m_s \right. \\ \left. + 200 \cdot C_{\alpha R} \cdot b \cdot K_{SBRF} \cdot u(t)^2 \cdot h_{roll} \cdot m_s \cdot C_{\alpha F} + 200 \cdot a^2 \cdot k_t \cdot C_{\alpha F} \cdot C_{\alpha R} + 200 \cdot b^2 \cdot k_t \cdot C_{\alpha F} \cdot C_{\alpha R} - 100 \cdot C_{\alpha F} \cdot a \cdot u(t)^2 \cdot k_t \cdot m_c + 400 \cdot C_{\alpha F} \cdot a \cdot b \cdot k_t \cdot C_{\alpha R} + 100 \cdot C_{\alpha R} \cdot b \cdot u(t)^2 \cdot k_t \cdot m_c \right. \\ \left. - 1962 \cdot a^2 \cdot h_{roll} \cdot m_s \cdot C_{\alpha F} \cdot C_{\alpha R} - 1962 \cdot b^2 \cdot h_{roll} \cdot m_s \cdot C_{\alpha F} \cdot C_{\alpha R} + 200 \cdot K_{CBRF} \cdot u(t)^2 \cdot h_{roll} \cdot m_s \cdot a \cdot C_{\gamma F} \cdot C_{\alpha R} + 981 \cdot C_{\alpha F} \cdot a \cdot u(t)^2 \cdot h_{roll} \cdot m_c \cdot m_s - 3924 \cdot C_{\alpha F} \cdot a \cdot b \cdot h_{roll} \cdot m_s \cdot C_{\alpha R} \right. \\ \left. - 981 \cdot C_{\alpha R} \cdot b \cdot u(t)^2 \cdot h_{roll} \cdot m_c \cdot m_s \right) :$$

Vehicle Parameters

$$\text{CarParams} := [a = a_{CG}, b = b_{CG}, l_{wb} = L_{wb}, m_c = m_{curb}, m_s = m_{sprung}, h_{roll} = h_{CG} - h_{RC}, I_{zz} = I_z, I_{xx} = I_x, c_t = C_r, k_t = K_t]$$

$$[a = 1.2, b = 1.3, l_{wb} = 2.5, m_c = 1000, m_s = 900, h_{roll} = 0.4, I_{zz} = 2000, I_{xx} = 750, c_t = 1521.000000, k_t = 54118.04372] \quad (1.1)$$

$$\text{TireParams} := [C_{\alpha F} = 2 \cdot C_{\alpha}, C_{\alpha R} = 2 \cdot C_{\alpha}, C_{\gamma F} = 2 \cdot C_{\gamma}, C_{\gamma R} = 2 \cdot C_{\gamma}]$$

$$[C_{\alpha F} = 50000, C_{\alpha R} = 50000, C_{\gamma F} = 10000, C_{\gamma R} = 10000] \quad (1.2)$$

Neutral-Steer Response

The vehicle's steering response is determined to be "neutral" when $K_{us3DOF} = 0$, so keeping all but one of the roll parameters constant, it is possible to determine the value of a roll parameter (in this case, K_{SBRF}) to provide a neutral response. This can then be used to plot with other steering responses to provide a guideline as to how far from neutral the response is.

Value of K_{SBRF} at neutral steer...

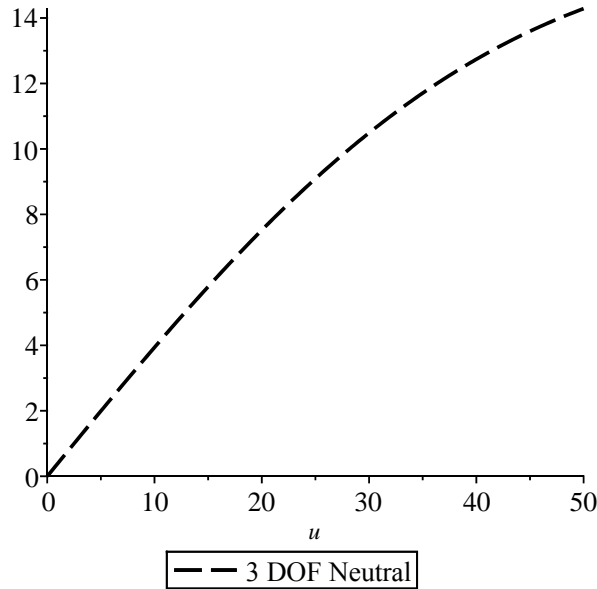
$$K_{NeutralSBRF} := \text{solve}(\text{eval}(K_{us3DOF} = 0, [\text{op}(\text{CarParams}), \text{op}(\text{TireParams}), K_{SBRF} = 0.1, K_{CBRF} = 1, K_{CBRR} = 0.1]), K_{SBRF})$$

$$-0.07998875854 \quad (1)$$

$$\text{NeutralParams} := [K_{SCBRF} = 0, K_{SCBRR} = 0, K_{SBRF} = 0.1, K_{CBRF} = 1, K_{CBRR} = 0.1, K_{SBRF} = K_{NeutralSBRF}]$$

$$[K_{SCBRF} = 0, K_{SCBRR} = 0, K_{SBRF} = 0.1, K_{CBRF} = 1, K_{CBRR} = 0.1, K_{SBRF} = -0.07998875854] \quad (2)$$

$\text{YawGain3DOFNeutral} := \text{plot}(\text{eval}(\text{YawGain}, [\text{op}(\text{CarParams}), \text{op}(\text{TireParams}), \text{op}(\text{NeutralParams})]), u = 0..50, \text{colour} = \text{black}, \text{thickness} = 2, \text{linestyle} = \text{dash}, \text{legend} = "3\text{ DOF Neutral}"):$



This interactive tool allows the user to try various combinations of steer- and camber-by-roll coefficients and observe the effect on the yaw gain curve and the value of the understeer coefficient, K_{us} .

